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Brief communication

Explosive flow of boiling and degassing liquids out of pipes and reservoirs: the influence of wall friction

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1. Introduction

The sudden depressurisation and the rapid flow of a liquid through a break in a pipe is a dramatic situation that may happen not only in nuclear plants (loss of coolant accident) but also in pipelines filled with readily boiling hydrocarbons and chemical plants filled with a gas-saturated liquid. Special features of such explosive flows come about when the pressure decreases below the saturation value. The resulting bubbly mixture happens to have an anomalously low velocity of sound and the discharge of boiling or degassing liquids is always accompanied by important gas-dynamic effects. In most situations, the pipe has a short length, the discharge is very quick, and the phenomena are dominated by the inertia of the mixture and quasi-adiabatic processes in the bubbles. However, for longer channels (such as those met in petroleum industry) this first and well-known regime (Edwards and O'Brien, 1970; Gubaidullin and Ivandeev, 1978; Shagapov, 1979; Nigmatulin, 1991; Nigmatulin and Soplentkov, 1994) is followed by a second one in which the dominant role is played by the wall friction and the bubbly mixture evolves under saturated conditions. Since the volumetric content of the gaseous phase is steadily increasing in the discharge process, a third regime can happen in very long pipes, in which the bubbly mixture is replaced by a slug-liquid mixture or even by a droplet-gas mixture. The model to be presented hereafter concerns the second regime only, and is limited to bubbles volume fractions less than 0.2 (approximately). In Section 2 we show that the bubbly mixture can be represented by a simple equation of state in

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the barotropic form $\rho(p)$. The relevant mass and momentum equations are obtained in Section 3 and they result in a single non-linear equation describing the evolution of the pressure with space and time. In Section 4 we find the pressure profiles in the broken pipe as well as the discharge outflow as a function of time. When the pipe is connected to a reservoir, the discharge process ends with a stationary outflow calculated in Section 5.

2. Equation of state for close-to-saturation bubbly mixtures

2.1. Vapour–liquid mixtures

The mean mass ρ per unit volume of the mixture can be written as $1/\rho = x/\rho_G^\circ + (1-x)/\rho_L^\circ$ where x is the mass-fraction of the vapour phase, while ρ_L° and ρ_G° are the true mass densities of liquid and vapour, respectively. ρ_L° is a function of the average pressure and temperature of the liquid phase, which are generally different from the average pressure and temperature of the vapour phase, the two variables on which ρ_G° depends. This suggests that ρ is generally a complicated function of a large number of variables. In this section, we will briefly describe the conditions to be met for the mean density of the mixture to be a function of the average pressure p only, and we will obtain an explicit expression for the equation of state $\rho(p)$.

The average pressures of the two phases are generally different because, in the situations to be described, the vapour bubbles are expanding. However, provided we are not interested in the very first stage of bubble expansion, that difference is very small when compared to the mean pressure of the mixture. Similarly, a temperature difference always exists between the two phases, because the vapour is produced under saturation conditions, while the liquid is likely to be in a superheated state during the depressurization process. However, if we discard the initial stage of bubble growth, the amount of interfaces per unit volume will be important, the energy exchanges between the two phases will be efficient, and the difference between the two average temperatures will be small when compared to the mean temperature T of the mixture. Lastly, despite the fact that bubbles move with an average velocity different from that of the fluid, the relative velocity is much smaller than the mixture velocity in the discharge process. To sum up, a single velocity, a single temperature and a single pressure are enough to describe the bubbly mixture in the second regime. Moreover, the thermodynamic state of the mixture will always keep close to saturation conditions, so that T is connected to p as $T = T_s(p)$. Then, the Clausius–Clapeyron relation can be used to obtain $\rho_L^\circ/\rho = 1 + (\rho_L^\circ x \cdot \ell/T_s) dT_s/dp$, where $\ell(p)$ is the specific heat of evaporation. At this point, ρ is a function of p and x . To proceed further, let us consider the specific enthalpy of the mixture which can be written as

$$i = i_L + x(i_G - i_L) = i_0 + c_L(T_s(p) - T_0) + x \cdot \ell(p), \quad (2.1)$$

where c_L is the specific heat of the liquid and i_0 is the specific enthalpy of the liquid in the initial state ($p_0, T_0 = T_s(p_0)$). The evolution of i is described by the energy balance of the mixture. The change of enthalpy due to the pressure drop and the increasing vapour mass fraction is much larger than the heat exchanges through the walls of the pipe and also much

larger than the viscous heat production. As a consequence, the energy balance is reduced to its simple non-dissipative form $\rho di/dt = dp/dt$, or $\rho \partial i / \partial p|_x dp/dt + \rho \partial i / \partial x|_p dx/dt = dp/dt$ since i is a function of p and x only. Because our model holds for a bubbly mixture for which $\rho \gg \rho_G^\circ$, one deduces $\rho \partial i / \partial p|_x \cong \rho c_L dT_s/dp \cong (\rho/\rho_G^\circ)(c_L T_s/\ell) \gg 1$. As a consequence, the energy balance amounts to the remarkably simple result

$$di/dt \cong 0 \quad \text{or} \quad i \cong i_0. \quad (2.2)$$

According to Eqs. (2.1) and (2.2), the mass fraction is a function of pressure given by

$$x = c_L(T_0 - T_s)/\ell. \quad (2.3)$$

It is clear that this result holds at any location in the flow where $p < p_0$ and that $x = 0$ when $p = p_0$. Substituting that result into the above expression for the mass density ρ , one finds the bubbly mixture to be depicted by a rather simple equation of state $\rho(p)$ which is given in explicit form by

$$\rho_L^\circ/\rho = 1 + \rho_L^\circ c_L(T_0/T_s - 1)dT_s/dp. \quad (2.4)$$

If we neglect the compressibility of the liquid phase, the velocity of sound in the mixture is

$$C^{-2} = \rho^2 c_L T_s^{-2} [T_0(dT_s/dp)^2 - T_s(T_0 - T_s)d^2 T_s/dp^2]. \quad (2.5)$$

For future reference, it is interesting to introduce C_0 , the velocity of sound for p close to the initial pressure p_0 . In this case the second term on the right-hand side is negligible and one obtains $C_0 = (\rho_{G_0}^\circ/\rho_L^\circ)\ell_0(c_L T_0)^{-1/2}$, where ℓ_0 and $\rho_{G_0}^\circ$ are the heat of evaporation and vapour density at pressure p_0 . It happens that in a rather large pressure domain (far from the critical point generally) the pressure dependence of ℓ can be neglected, and the saturation temperature can be represented accurately as

$$T_s(p) = T^*/\text{Log}(p^*/p) \quad (2.6)$$

where T^* and p^* are empirical parameters. As a consequence, both ρ/ρ_L° and C/C_0 depend on the ratio p/p_0 only, a result that will prove important later on. We checked that the exact numerical values extracted from tables of data for bubbly-water and bubbly-propane, are well fitted by the approximate expressions (2.4), (2.5) and (2.6) in a large pressure and temperature range, not too close to the critical point region.

2.2. Gas–liquid mixtures

The bubbly mixture is now made of a liquid containing some amount of dissolved gas and of bubbles filled with the same gas. The liquid is assumed to be cold enough for its vapour to have a negligible presence in the bubbles. For mostly the same reasons as for liquid–vapour mixtures, we will neglect any difference between the average velocities, temperatures and pressures of the two phases. At variance with liquid–vapour mixtures, the temperature of the gas–liquid mixture will be supposed to be a constant, T_0 . Finally, to use Henri's law in non-equilibrium situations, we will suppose that the number of bubbles is large enough for the

diffusion of dissolved gas towards growing bubbles to be quasi-instantaneous. The mean density of the mixture is written as $\rho = \varepsilon\rho_G^\circ + (1 - \varepsilon)\rho_L^\circ$ where ε is the volume fraction of the bubbles, while ρ_G° and ρ_L° are the true densities of the gas and liquid, respectively. The mass fraction of the gas dissolved in the liquid is noted by k and this mass fraction is some function of the liquid pressure. Since the gas exists as dissolved in the liquid or filling the bubbles, the mass of gas per unit volume of the mixture is $\rho_G = \varepsilon\rho_G^\circ + k(1 - \varepsilon)\rho_L^\circ$. In the absence of any significant relative motion between the two phases, the mass fraction ρ_G/ρ is a constant equal to the mass fraction k_0 at pressure p_0 . One deduces that the volume fraction of the bubbles is a function of pressure given by

$$\varepsilon = (k_0 - k)\rho_L^\circ [(k_0 - k)\rho_L^\circ + (1 - k_0)\rho_G^\circ]^{-1}. \quad (2.7)$$

This result corresponds to Eq. (2.3) for liquid–vapour mixtures. It is now clear that the mean density of the mixture is a function of the mean pressure only, given by $(1 - k)/\rho = (1 - k_0)/\rho_L^\circ + (k_0 - k)/\rho_G^\circ$. We will henceforth limit ourselves to cases in which the initial mass fraction of dissolved gas is very weak ($k_0 \ll 1$). Then, k is simply proportional to p and one can write (Henri's law) $k/k_0 = p/p_0$. If the gas in the bubbles behaves like an ideal gas, the equation of state becomes

$$\rho_L^\circ/\rho = 1 + K_0(p_0/p - 1), \quad (2.8)$$

(compare with Eq. (2.4)) and the related velocity of sound is (compare with Eq. (2.5))

$$C/C_0 = (p/p_0)(\rho_L^\circ/\rho). \quad (2.9)$$

In these expressions, C_0 is the velocity of sound when the pressure of the mixture is close to p_0 and K_0 is a dimensionless parameter. They are respectively defined as $C_0 = (p_0/K_0\rho_L^\circ)^{1/2}$ and $K_0 = k_0(\rho_L^\circ/\rho_{G_0}^\circ)$ where $\rho_{G_0}^\circ$ stands for the density of the gas at pressure p_0 . Note that for gas–liquid mixtures (like for the vapour–liquid mixtures considered above), C/C_0 and ρ/ρ_L° are functions of p/p_0 only. A liquid containing a foreign gas is said to be ‘ideal’ when $K_0 = 1$. Such is the case of water saturated with carbon dioxide around $T = 288$ K. In this case, the equation of state and the sound velocity have particularly simple expressions since $\rho/\rho_L^\circ = p/p_0$ and $C = C_0 = (p_0/\rho_L^\circ)^{1/2}$.

3. Mass and momentum equations

After the initial regime of nucleation and rapid growth, the bubbly mixture reaches a second regime in which, as seen in the previous section, the differences of pressure, temperature and velocity between the two phases can be neglected. Such a boiling or degassing mixture may be described by a barotropic equation of state $\rho(p)$ which already takes the balance of energy into account. Then, the evolution of the mixture is completely described by a single mass balance and a single momentum balance. If the evolution occurs in a long pipe, it is advisable — at least as a first step — to consider a one-dimensional description of the flow involving variables that are averaged over the cross-section and to write the mass and momentum balance as

$$\partial\rho/\partial t + \partial\rho w/\partial z = 0, \quad (3.1)$$

and

$$\rho(\partial w/\partial t + w\partial w/\partial z) = -\partial p/\partial z - \tau(w) \quad (3.2)$$

where τ represents the average friction force per unit length of the pipe and w is the axial velocity of the mixture. Although we have unsteady flows in mind, we will use expressions for the friction force that were established for steady situations. We admittedly suppose that the characteristic time t^* of the unsteady flow is considerably less than the typical discharge time. That approximation can be justified along the lines sketched below (see Eqs. (3.5) and (3.6)). The expression of the friction force depends on the Reynolds number of the flow, and we will distinguish between a linear and a quadratic law of resistance. This choice may appear somewhat arbitrary but our main aim is to present two extreme cases of friction which, we hope, will reveal some general features concerning discharges in pipes.

The first friction law is relevant for pipes with relatively small cross-sections and it appears as

$$\tau = \rho w/t_w \quad \text{with } t_w = R^2/8\nu_L \quad (3.3)$$

where R is the pipe radius and ν_L is the kinematic viscosity of the liquid. That expression for t_w holds provided the vapour or gas volume fraction is small. If not, the relaxation time will depend on the volume fraction. This complication is not necessary in the situations considered hereafter which concern volume fractions never exceeding 0.2 (approximately).

The second friction law is suited for larger pipes and is expressed as

$$\tau = \rho|w|w/z_w \quad \text{with } z_w = R/\lambda \quad (3.4)$$

where λ is a numerical coefficient of order 10^{-2} , possibly depending on the roughness of the pipe wall. In the momentum equation (3.2), the friction force competes with the acceleration force. We now specify the conditions to be met for $\tau(w)$ to play a dominant role. For the flow of a compressible mixture in a pipe, the velocity of sound C provides an estimate for the spatial variations of the velocity. It is not difficult to see that the acceleration term is negligible provided disturbances develop in the pipe with characteristic times and distances much larger than t^* and z^* defined as

$$t^* = t_w \quad \text{and} \quad z^* = Ct_w \quad \text{for the linear friction law} \quad (3.5)$$

$$t^* = z_w/C \quad \text{and} \quad z^* = z_w \quad \text{for the quadratic friction law.} \quad (3.6)$$

For example, in the problem of a sudden depressurization at some point of a long channel, the inertial forces will be negligible after time t^* and at distances larger than z^* from the break point. In that case, the mass and momentum balances can be gathered into a single equation describing the evolution of pressure

$$\partial p/\partial t = C^2 t_w \partial^2 p/\partial z^2 \quad \text{and} \quad \rho w = -t_w \partial p/\partial z \quad (3.7)$$

or

$$\partial p / \partial t = C^2 z_w^{1/2} \partial(\rho \partial p / \partial z)^{1/2} / \partial z \quad \text{and} \quad \rho |w| w = -z_w \partial p / \partial z \quad (3.8)$$

for the linear and the quadratic friction force, respectively. With a different friction law $\tau(w)$, a different spatial derivative would have been found in the evolution equation for the pressure. When limiting ourselves to the two particular cases Eqs. (3.7) and (3.8), we have in mind to provide two specific but representative examples.

4. Main features of the discharge in long pipes

Let us consider a long pipe initially filled with a motionless liquid at pressure p_0 and temperature T_0 . The pipe is suddenly broken at some point where the pressure is reduced to an external pressure $p_e < p_0$. The liquid begins to flow out of the break and the process of boiling or degassing is initiated. After a short initial stage, the two-phase mixture is described by Eqs. (3.7a) or (3.8a) associated with the boundary and initial conditions

$$p = p_0(t = 0, z \geq 0) \quad \text{and} \quad p = p_e(t \geq 0, z = 0). \quad (4.1)$$

According to what was found in Section 2, the saturated bubbly mixture is characterized by a density and a velocity of sound such that ρ/ρ_L° and C/C_0 depend on the pressure ratio p/p_0 only. It is then not difficult to convince oneself that the pressure field $p(z, t)$, solution of Eqs. (3.7a) or (3.8a), displays a *self-similar* profile $p(Z)$ with $Z = z/C_0(t_w t)^{1/2}$ for the linear friction force and $Z = z/C_0(C_0 z_w \rho_L^\circ / p_0)^{1/3} t^{2/3}$ for the quadratic friction force. More generally, there is a one-to-one correspondence between $p(Z)$ and $\tau(w)$, and the experimental results concerning the propagation of pressure along the pipe can give an insight into the w -dependence of the wall friction force.

There is a quantity which is still easier to measure experimentally, namely the rate of mass flowing out of the break, defined as $q = -\rho w|_{z=0}$. According to Eqs. (3.7b) and (3.8b), the mass outflow decreases in time as

$$q = t_w \partial p / \partial z|_{z=0} = \chi_L(p_0/C_0)(1 - p_e/p_0)(t_w/t)^{1/2} \quad (4.2)$$

for the linear friction force and

$$q = (z_w \rho \partial p / \partial z|_{z=0})^{1/2} = \chi_Q(p_0/C_0)(1 - p_e/p_0)^{2/3} (C_0 z_w \rho_L^\circ / p_0 t)^{1/3} \quad (4.3)$$

for the quadratic friction force. Here χ_L and χ_Q are two scalars with values of order 1, functions of the pressure ratio p_e/p_0 only. We conclude that the decrease of $q(t)$ obeys a power law with an exponent in the range between $-1/3$ and $-1/2$. This range is not very broad but there is again a one-to-one correspondence between the characteristic exponent and the wall friction $\tau(w)$.

According to standard results of compressible fluid dynamics (Landau and Lifshitz, 1959, all that has been deduced above is correct provided the external pressure p_e is not too low as compared to the initial pressure in the pipe p_0 , and more precisely, if it exceeds a lower value p_c , depending on p_0 and defined as

$$C(p_c) = \int_{p_c}^{p_0} [\rho(p)C(p)]^{-1} dp \quad (4.4)$$

where C is the velocity of sound. For boiling water with initial pressure $p_0 = 2.5 \times 10^6$ Pa, the solution of the above relation is $p_c = 1.7 \times 10^5$ Pa. For a degassing liquid described by Eqs. (2.8) and (2.9), one can deduce p_c from the simple equation $(1 + \log P_c) / P_c = 1 - 1/K_0$, where $P_c = p_c/p_0$. For example, $P_c = 1/e$ for $K_0 = 1$ and $P_c = 0.44$ for $K_0 = 1.7$. When the pressure at the break decreases to p_c , the outflow is no longer described by Eq. (4.2) or Eq. (4.3), but reaches a stationary ‘sonic’ value to be described at the end of the next section.

5. Steady discharge out of a reservoir through a pipe

In the previous section, the finite length of the pipe was not taken into account and the bubbly liquid outflow was steadily decreasing in time. A more usual situation is when the pipe ends into a large reservoir, located at a distance L from the break point and with initial pressure p_0 . The non-stationary process depicted above will finally merge into a stationary discharge q_0 provided by the reservoir. In the pipe itself, the stationary outflow is a solution of the mass and momentum balances (3.1) and (3.2) written as

$$\rho w = q_0 \quad (5.1)$$

$$d(q_0^2/\rho + p)/dz = -\tau(w). \quad (5.2)$$

When explicit expressions are provided for the wall friction force and for the equation of state $\rho(p)$, one can deduce q_0 as a function of three parameters : the pipe length L , the external pressure p_e and the pressure p_i at the inlet, i.e. at the junction of the pipe with the reservoir. As an example, for gas–liquid mixtures with $K_0 = 1$ and a quadratic friction law, one obtains

$$q_0^2 = (p_i^2 - p_e^2)/2C_0^2[\log(p_i/p_e) + L/z_w].$$

Details will not be given for other cases since there is no special difficulty in solving Eqs. (5.1) and (5.2). All that can be said is that the dependence of q_0 on $p_i - p_e$ is generally far from linear. The next problem is to calculate p_i as a function of the initial pressure p_0 in the pipe and the reservoir. When the steady flow is due to the emptying of a huge reservoir with a free-surface at pressure p_0 , the velocity at the inlet can be estimated by Bernoulli’s relation as

$$q_0^2/2\rho_i^2 = \int_{p_i}^{p_0} (1/\rho)dp. \quad (5.3)$$

A solution can be found for the steady flow out of the reservoir provided one can find an inlet pressure p_i verifying $p_e < p_i < p_0$ and satisfying Eq. (5.3). Moreover, for the above results to hold, the external pressure p_e must be larger than the pressure p_c defined at the end of Section 4. If p_e happens to be lower than p_c , the outlet pressure becomes p_c , the outlet velocity is the velocity of sound $C(p_c)$ and the mass flow rate saturates at the maximum value $q_0 = \rho(p_c)C(p_c)$.

6. Conclusion

We have given a brief description of a particular stage of the explosive flow of boiling or degassing liquids. The main focus was on the so-called second regime, where the bubbly mixture can be represented by a one-velocity, one-pressure and one-temperature compressible fluid. We strived to give model expressions for the equation of state, as well as complete analytical solutions for the bubbly mixture motion. In particular, we analysed the non-stationary discharge out of a long pipe. The self-similar pressure profiles and the decrease of the outflow as a power of time are some of the interesting analytical results of our 1D model that could be observed in future experiments, or in future numerical simulations based on a full 3D model for the discharge flow.

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